Embedded graphs

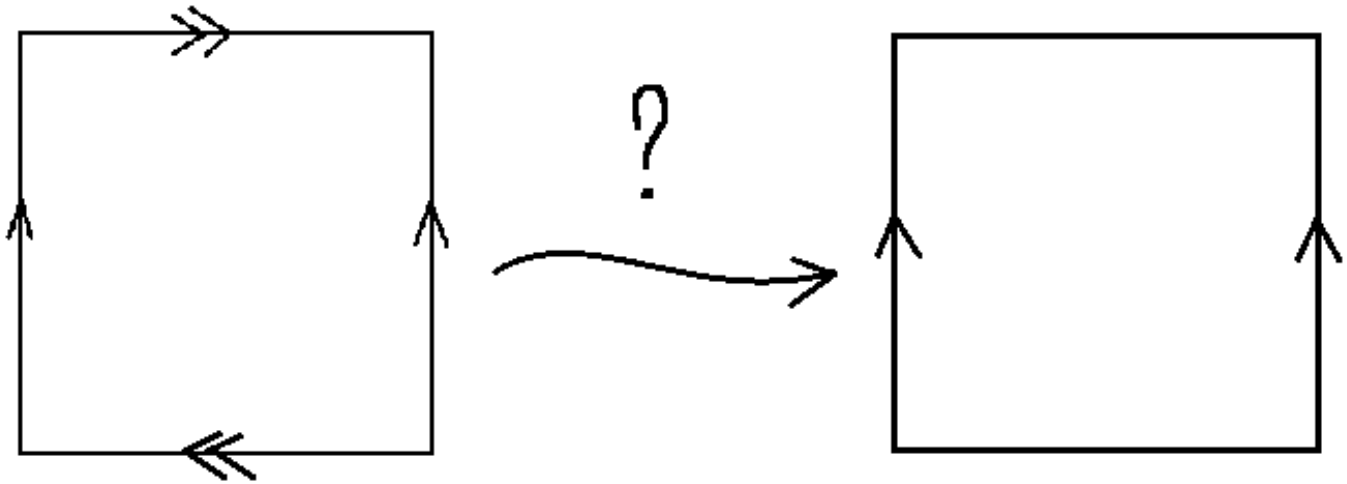
Sasha Patotski

Cornell University

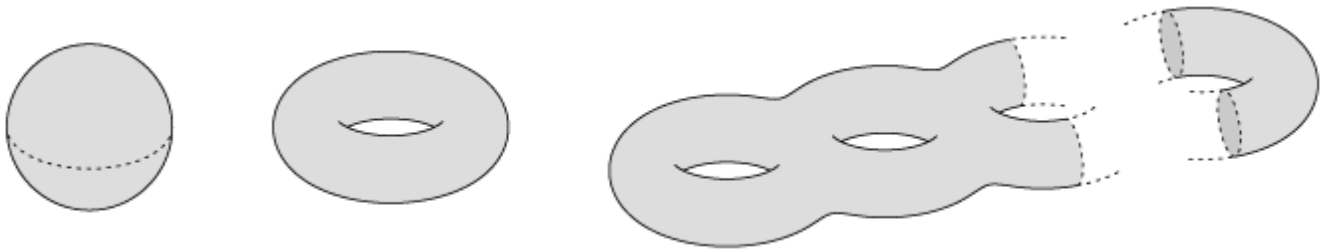
ap744@cornell.edu

November 20, 2014

Can you cut a Klein bottle into a single Möbius band?



How to get a sphere with g handles out of a polygon?



Non-planar graphs K_5 and $K_{3,3}$

Assume Euler formula: For any planar graph G one has $V - E + F = 2$.

Non-planar graphs K_5 and $K_{3,3}$

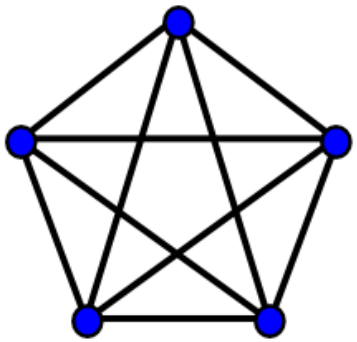
Assume Euler formula: For any planar graph G one has $V - E + F = 2$.

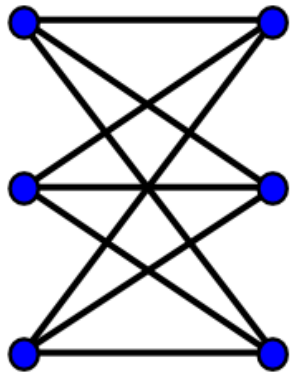
Prove that K_5 and $K_{3,3}$ are not planar.

Non-planar graphs K_5 and $K_{3,3}$

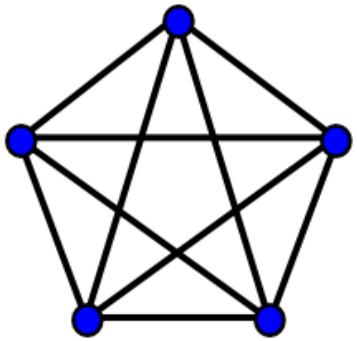
Assume Euler formula: For any planar graph G one has $V - E + F = 2$.

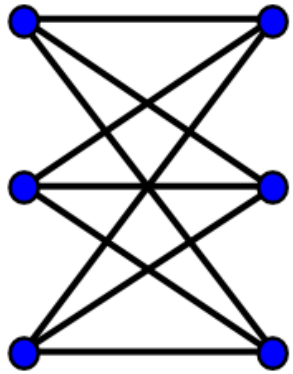
Prove that K_5 and $K_{3,3}$ are not planar.

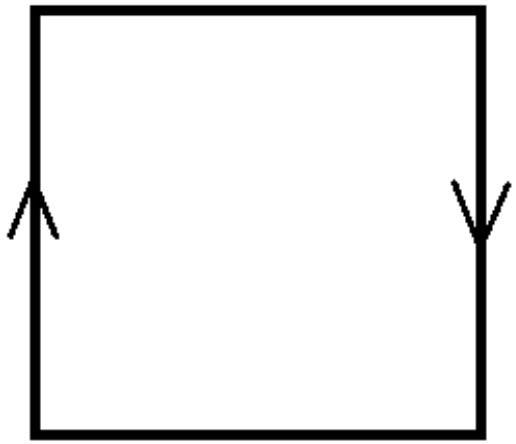




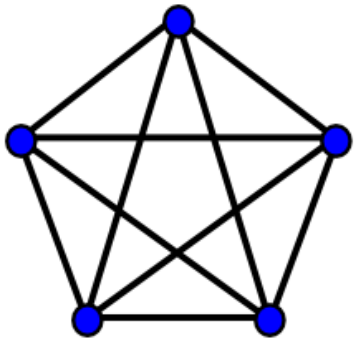
Can K_5 and $K_{3,3}$ be embedded into a Möbius band?

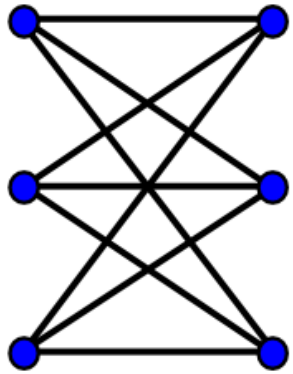


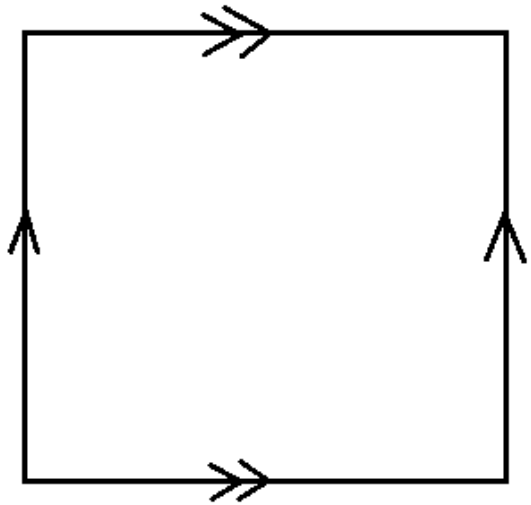


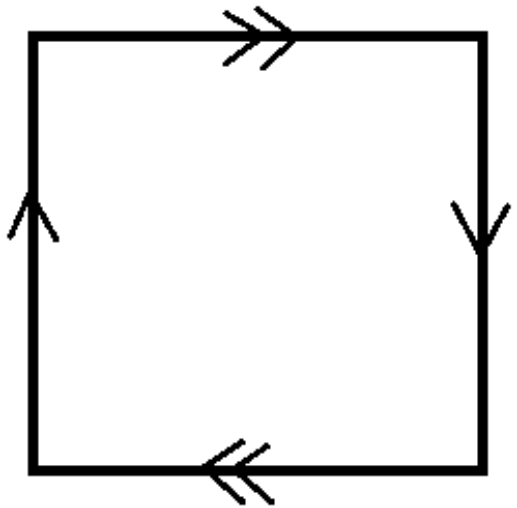


Embed K_5 into torus and $K_{3,3}$ into $\mathbb{R}P^2$



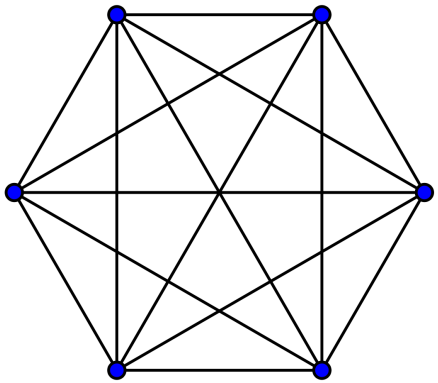


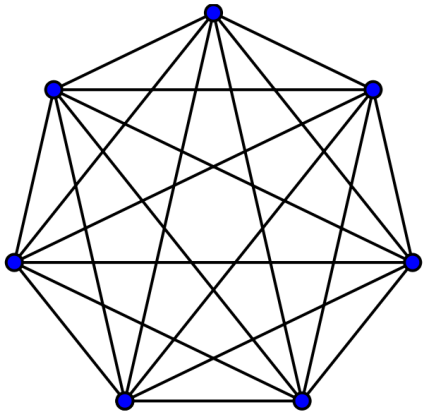




Exercise

Can we embed K_6 into a torus?



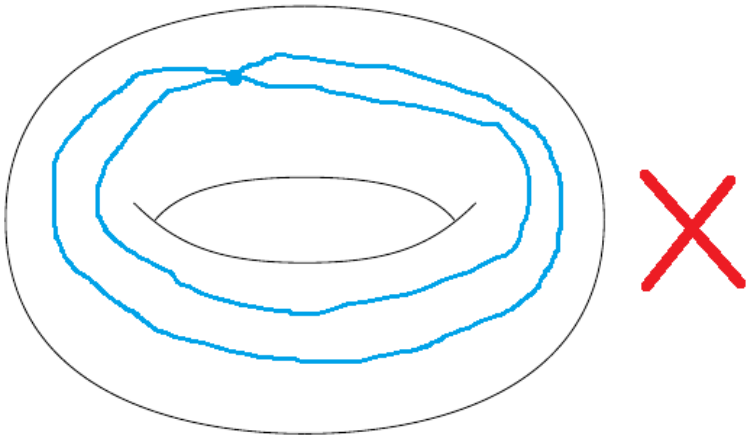


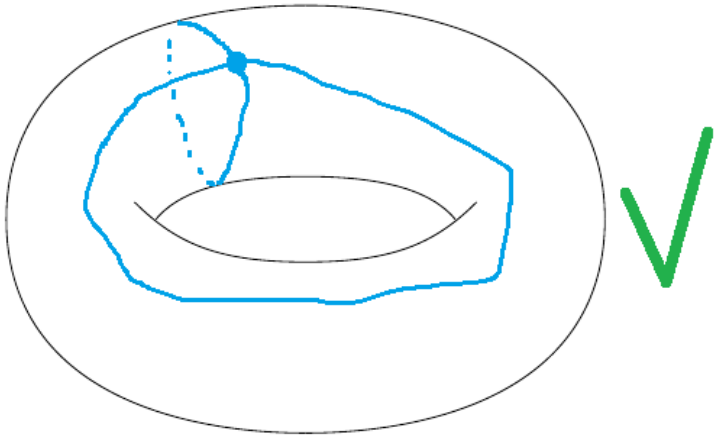
Can we embed K_7? or K_8? what about K_9?

2-cell embeddings of graphs

Definition

A 2-*cell embedding* of a graph G into a surface Σ is such an embedding, so that after you remove the graph, the resulting surface will be a disjoint union of discs.





Euler formula

Theorem

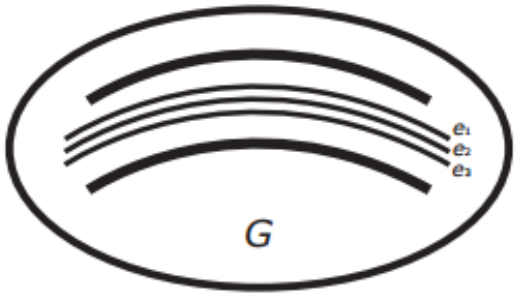
Suppose graph G is 2-cell embedded into a sphere with g handles. Then

$$V - E + F = 2 - 2g,$$

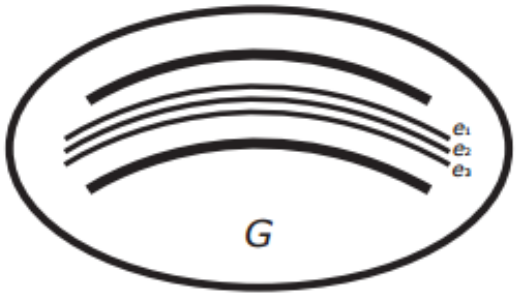
where V is the number of vertices, E is the number of edges and F is the number of faces.

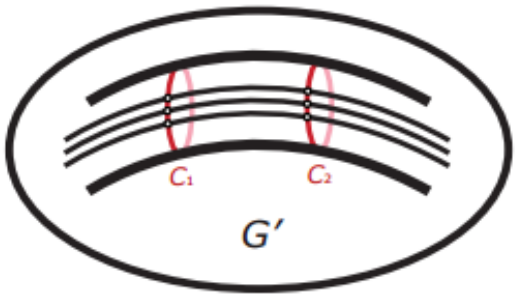
Induction on g. If $g = 0$, then we are in the case S^2. Here, induction on edges.

End of the proof

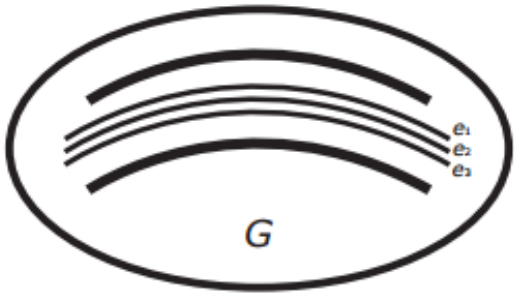


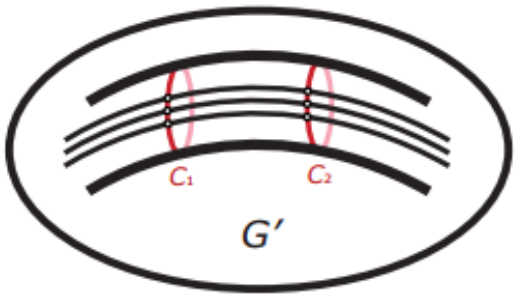
End of the proof

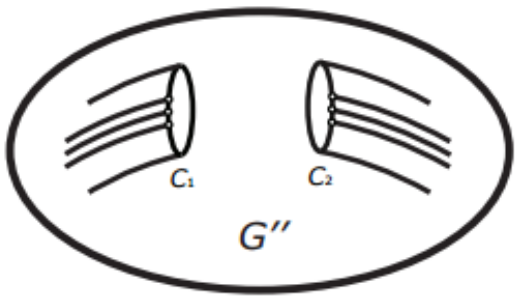




End of the proof







Application

Can we embed K_8 into a torus?

Application

Can we embed K_8 into a torus?

Theorem

For any graph there exist a sphere with g handles where it can be embedded.